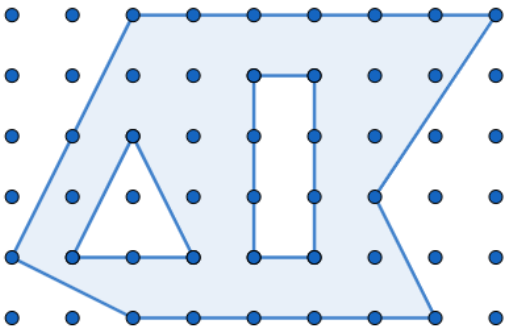
Extract

Investigation on the Generalization of Pick's Formula



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Part 1: Introduction

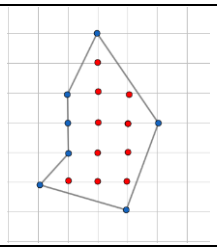
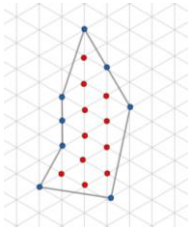
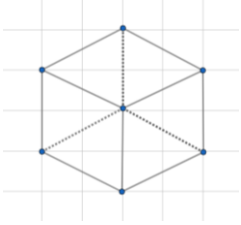
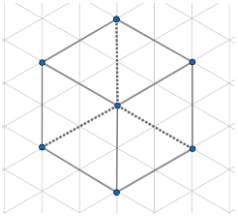
a) Motivation

In many situations, we may face problems in finding the area of polygons. We have learnt some ways to tackle the problems using splitting method and fill-in method. However, when we meet some difficult polygons, it may be too complex to find their area just by the two methods stated above.

Interestingly, we discover that there is a simple and efficient way to find the area of polygon in square lattice system, that is, **Pick's formula**. The result of Pick's formula was first described by a mathematician Georg Alexander Pick back in 1899. Owing to the simplicity of Pick's formula, we are motivated to go deeper on the power of Pick's formula in finding area in different kind of shapes, and for both area of 2D polygons and total surface area of 3D solids.

Our investigations in this project intend to systematically prove the original Pick's formula work on square lattice system, Pick's formula can be generalized on isometric lattice system, and the exploration of Pick's formula in finding the total surface area of 3D solids with special focus on platonic solids and regular pyramids.

The table below shows examples on both lattice systems. The area of a 2D polygon on square lattice system and isometric grid system is shown. For the 2D polygon, blue dots represent boundary points and red dots represent interior points. The projection of a 3D cube on the two systems also demonstrated in the table. The projection arouses our interest in the application of Pick's formula in finding the total surface area of platonic solids.

	Square lattice system	Isometric lattice system
2D Polygon on different lattice systems		
3D Cube projection on different lattice systems		

Here, we introduce Pick's formula in square lattice system:

$$A = \frac{1}{2}B + I - 1$$

A is the area, B is the number of boundary lattice points, and I is the number of interior lattice points

b) Problem Statements

1a: Proving Pick's formula for area of rectangles on square lattice system.
1b: Proving Pick's formula for area of equilateral triangles on isometric lattice system.
2: Proving Pick's formula for area of elementary triangles on both square lattice system and isometric lattice system.
3: Proving Pick's formula for area of generally combined polygons on both systems.
4: Proving Pick's formula applying to polygons with non-touching holes and all of its vertices on lattice point.
5a: Applying Pick's formula in finding total surface area of platonic solids.
The diagram below shows the platonic solids under investigation.

5b: Applying Pick's formula in finding total surface area of regular pyramids.

Part 7: Summary and Insight on Applications

Having proved the Pick's formula work on different kinds of polygon, from the simplest form to a more complex form, we have a simple but powerful tool to find the area of polygons.

We note that the generalized Pick's formula for polygon with non-touching holes is

$$A_2 = I_2 + \frac{B_2}{2} - 1 + n$$

where n is the number of holes in the subtracted polygon.

When we do further research on the isometric projection of solids, we find the crystallography is a discipline that deserves further investigation on the application of Pick's formula. With the proved result on Pick's formula and deeper understanding of two different lattice systems, we are curios and positive on such further investigation.

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